Available online at www.ijpab.com

DOI: http://dx.doi.org/10.18782/2320-7051.6075

ISSN: 2320 – 7051

Int. J. Pure App. Biosci. **6 (1):** 52-55 (2018)

Research Article

International Journal of Pure & Applied Bioscience

OPEN ACCESS

Mathematical Characterization of Growth of a Local Landrace of Sorghum from Saudi Arabia

Fethi Soltani[1], Habib Khemira[2*], Sami Atifi[1] and Qassim Ghazwani[1]

[1]Department of Mathematics, Faculty of Science, Jazan University, Jazan, Saudi Arabia

[2]Centre for Environmental Research & Studies (CERS), Jazan University, Jazan, Saudi Arabia

*Corresponding Author E-mail: habibkhemira@yahoo.com

Received: 6.12.2017 | Revised: 15.01.2018 | Accepted: 19.01.2018

ABSTRACT

We develop a generalized sigmoidal model and we used it to describe the growth of a local tall landrace of sorghum from Jazan region of Saudi Arabia. This sorghum is grown year-around for fodder and in early fall for grain. The plants were grown in 80-L pots under near-ideal conditions. Heading occurred 62 days after planting (DAP), while physiological maturity was reached after 105 DAP. Evolution of plant growth as a function of time was best represented by the sigmoid model $\dot{y} = ky(1-\frac{y}{a})$, *where* $a = 251.11006$ *is the maximum height that the plant can theoretically reach and* $k = 0.00234$ *represents the cultivar's typical growth rate. It appears that this local sorghum landrace is a short-season cultivar given its short life cycle. However, this may vary slightly by location and year since the actual plant growth rate depends on several environmental factors such as soil fertility, ambient temperature, moisture availability, planting density and prevalence of pests.*

Key words: *Sigmoidal model, Sorghum, Phenological Stages.*

INTRODUCTION

Modeling is a symbolic representation of some aspects of an object or phenomenon of the real world[1]. The process of developing a model includes three major tasks: finding and formulating new questions, problematization and generation of data, and defining actions and studying their consequences. The best known models are those based on mathematics and deal with variables and parameters taking real values[2]. Modeling is increasingly used in all major fields of science and technology. In agriculture, it is widely used to predict plant behavior in order to plan different cultural operations such as sowing, planting, weed and pest control and harvest[3]. For instance, several mathematical models were developed to describe growth and development of maize[4], barley[5], soybean[6] and wheat[7]. These models vary in complexity depending on the number of independent variables considered in order to take into account the many factors that can influence plant growth such as temperature, water availability, soil fertility, fertilizer supply, crop genetic traits and pest pressure.

Cite this article: Soltani, F., Khemira, H., Atifi, S. and Ghazwani, Q., Mathematical characterization of growth of a local landrace of sorghum from Saudi Arabia., *Int. J. Pure App. Biosci.* **6(1):** 52-55 (2018). doi: http://dx.doi.org/10.18782/2320-7051.6075

The goal of this study was to develop a simplified mathematical dynamic model which can be used to describe the growth of a local landrace of red sorghum from Jazan region of Saudi Arabia. This sorghum is a tall landrace which is grown year-around for fodder and in early fall for grain (because of short photoperiod requirement). The plants can be cut several times before being allowed to flower and produce grain.

MATERIALS AND METHODS

Development of a dynamic model: In this study and for the purpose of practicality, we developed a dynamic model describing the growth of a local sorghum landrace as it relates to plant age. We introduced the following generalized sigmoid model:

$$\dot{y} = ky\left(1-(\frac{y}{a})^b\right),$$

where k is the relative growth rate coefficient, a is the maximum observable value of y and b is a constant that allows the shape of the sigmoid curve to be varied. This model derives from the exponential model for plant growth:

$$\dot{y} = yf(y),$$

where $f(y)$ equals the relative growth rate of the plant, which decreases as y increases.

A complete quantitative analysis of sorghum growth was performed by solving exact governing equations. To describe the behavior of the model, we started by searching equilibrium points, studied the stability of these equilibrium points then established phase portrait and chronic curves.

Plant material: Seeds of a local land race of red sorghum (*Sorghum bicolor* (L.) Moench) from the region of Ahad-Almassarha, Jazan (Kingdom of Saudi Arabia) (N; W) were sown on 19 September 2016 directly in three cylindrical 80-L plastic pots filled with a sandy loam soil and placed on an experimental plot on Jazan University campus (16° 58'N; 42° 33'E). After germination, some of the seedlings were thinned out to leave only 5 plants per pot evenly spread out. The plants were watered every four days alternatively with tap water (EC 1.5 dS/m) and a modified Hoagland nutrient solution[8] .Plant height and number of leaves were recorded every 5 days until grain maturity. The plant phenological stages were identified according to Rao *et al.*[9].

RESULTS AND DISCUSSION

Model development: This model matches the sigmoidal model of plant growth which has been extensively studied[1,2,10,11]. For this model, we determined the solutions and the equilibrium points. Next we proved that the model is stable and has a single inflection point.Evolution of plant growth as a function of time was best represented by a sigmoid model; this model is defined by the differential equation

$\dot{y} = ky(1-\frac{y}{a})$, where $k = 0.00234$ and $a = 251.11006$. The constant a is the maximum height that the plant can theoretically reach, and the constant k represents the cultivar's typical growth rate and is a hereditary characteristic of the plant type.

The generalized sigmoidal model:

We consider the generalized sigmoid model defined by the differential equation:

$$\dot{y} = ky\left(1-(\frac{y}{a})^b\right), \quad y(0) = y_0, \quad (1)$$

where k, a and b are positive constants.

Theorem 1. The solutions of the non-linear model (1) are the sigmoid functions given by

$$y = a\left(1+\lambda e^{-kbx}\right)^{-\frac{1}{b}}, \quad (2)$$

where

$$\lambda = \frac{a^b}{y_0^b} - 1.$$

Proof. We have

$$\dot{y} = ky\left(1-(\frac{y}{a})^b\right) \Rightarrow \frac{a^b}{y(a^b-y^b)}dy = kdx$$

$$\Rightarrow \int\left(\frac{1}{y}+\frac{y^{b-1}}{a^b-y^b}\right)dy = \int kdx$$

$$\Rightarrow \ln\left|\frac{y^b}{a^b-y^b}\right| = kbx + c$$

$$\Rightarrow y = a\left(1+\lambda e^{-kbx}\right)^{-\frac{1}{b}},$$

where c is a constant and $\lambda = e^{-c}$.

Theorem 2. The solutions (2) satisfy the following properties:

(i) $y \to a$ as $x \to \infty$.

(ii) $\dot{y} = \frac{\lambda k}{a^b} y^{b+1} e^{-kbx} \geq 0$.

(iii) $\ddot{y} = k\dot{y}\left(1-(1+b)(\frac{y}{a})^b\right)$.

Proof. The results of this theorem are deduced by straightforward calculation.

We can conclude that:

1. The solutions (2) are increasing and a is the limiting capacity for it.
2. The equilibrium points are the solutions constants of the model, i.e., the solutions for that

$$\dot{y} = 0 \Rightarrow ky\left(1-(\frac{y}{a})^b\right) = 0 \Rightarrow y = 0, a.$$

Therefore this model has two equilibrium points $y = 0, a$.

3. The solutions of the model converge to the equilibrium point $y = a$. Thus, the sigmoid model (1) is stable.
4. The inflection points of the curves of the model are the points for which $\dot{y} \neq 0$ and $\ddot{y} = 0$.

By Theorem 2 (ii) the model (1) has a single inflection point $y_I = a(1+b)^{-\frac{1}{b}}$.

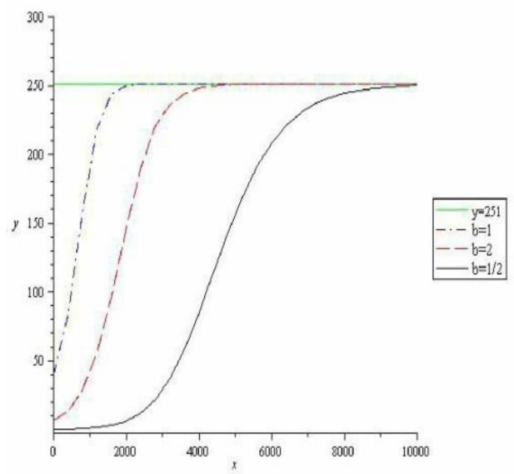

Fig. 1: Curves of the functions

$y = a\left(1+\lambda e^{-kbx}\right)^{-\frac{1}{b}}$, where $k = 2.10^{-3}$, $a = 251$, $\lambda = 39$ and for $b = 1$, $b = 2$ and $b = \frac{1}{2}$, respectively. These curves have an asymptote. That is $y = 258$.

Fitting of the model to the curve of local sorghum growth:

Phenology of local red sorghum: The seedlings emerged 3 days after planting (VE stage). The plants reached 5-leaf-stage 22 days after planting (DAP) (Fig. 2). Heading stage started after 62 DAP. The physiological maturity was reached 105 DAP. It appears that this landrace is a short-season cultivar given its short life cycle compare to other sorghum cultivars such as Barbare Red and Barbare White of Sudan[12]. However, this may vary slightly by location and year since the actual plant growth rate depends on several environmental factors such as soil fertility, ambient temperature, moisture availability, planting density and prevalence of pests[13].

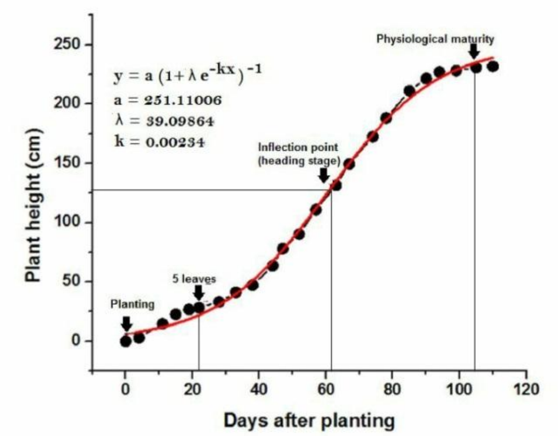

Fig. 2: Fitting of plant height against days after planting gave a sigmoidal curve

Fitting of the plant growth data: Growth data are represented by the sigmoid model (1), when $k = 0.00234$, $a = 251.11006$ and $b = 1$. The constant a is the maximum height that theoretically the plant can reach, and the constant k represents the rate of growth and is a hereditary characteristic of the plant type. This model has two equilibrium points $y = 0, a$. These two points represent sowing and maturity stages of the plant. The model is stable and has a single inflection point $y_I = \frac{a}{2} = 125.55503$. Biologically, this latter point represents the beginning of plant growth slowdown in preparation for flowering; i.e. the transition from vegetative growth to the reproduction phase (heading stage).

CONCLUSION

We develop a generalized sigmoidal model and we used it to describe the growth of a local tall landrace of sorghum from Jazan region of Saudi Arabia. Heading occurred 62 days after planting (DAP) while physiological maturity was reached after 105 DAP. Evolution of plant growth as a function of time was best represented by the sigmoid model $\dot{y} = ky(1 - \frac{y}{a})$, where $a = 251.11006$ is the maximum height that the plant can theoretically reach and $k = 0.00234$ represents the cultivar's typical growth rate. It appears that this local sorghum landrace is a short-season cultivar given its short life cycle. However, this may vary slightly by location and year since the actual plant growth rate depends on several environmental factors. The model should be further developed to take into account some of these environmental factors such as soil water and nutrient availability and pest pressure.

Acknowledgments

The authors thank the Deanship of Scientific Research of Jazan University for its financial support of this research project through the Future Scientists Program FS4-016.

REFERENCES

1. Mousset, S., Mathématiques Appliquées à la Biologie Modélisation des systèmes dynamiques. Université Claude Bernard, Lyon I, France, (2010).
2. Segel, L.A. and Edelstein-Keshet, L., A Primer in mathematical models in biology of other titles in applied mathematics, *SIAM*. **129:** 424 (2013).
3. Cao, W., White, J.W. and Wang, E., Crop Modeling and Decision Support. TsinghuaUniversity Press, Beijing, China, (2009).
4. Bert, F.E., Laciana, C.E., Podest, G.P., Satorre, E.H. and Men_endez, A.N., Sensitivity of CERES-Maize simulated yields to uncertainty in soil properties and daily solar radiation. *Agricultural Systems*, **94(2):** 141-150 (2007).
5. Juskiw, P.E. Jame, Y. and Kryzanowski, L., Phenological development of spring bar-ley in a short-season growing area. *Agronomy Journal* **93:** 370-379 (2001).
6. Pedersen, P., Boote, K.J., Jones, J.W. and Lauer, J.G., Modifying the CROPGRO-Soybean model to improve predictions for the Upper Midwest. *Agronomy Journal*, **96:** 556-564 (2004).
7. O'Leary, G.J., Connor, D.J. and White, D.H., A simulation model of the development, growth and yield of the wheat crop. *Agric. Syst.* **17:** 1-26 (1985).
8. Hoagland, D.R. and Arnon, D.I., water-culture method for growing plants without soil. Circular. *California Agricultural Experiment Station*, **347:** No. 2nd edition pp.32 (1950).
9. Rao, S.S., Elangovan, M., Umakanth, A.V. and Seetharama, N., Characterizing phenology of sorghum hybrids in relation to production management for high yields. NRCS-ICRISAT Learning Program on Sorghum Hybrids Parents and Hybrids Research and Development, 6 - 17 February 2007, NRCS and ICRISAT, Hyderabad (2007).
10. Blanchard, P., Devaney, R. and Hall, G., Diferential equations, 3rd ed., Thomson Brooks / Cole, Boston University (2006).
11. Teschl, G., Ordinary differential equations and dynamical systems. Providence: American Mathematical Society (2012).
12. Abdelrasool, Hanadi Osman, Growth and Productivity of Barbare, a long Day Sorghum {*Sorghum bicolor* (L) Moench} Genotype, During Winter and Summer in Two locations in Sudan. Ph.D. thesis The University of Khartoum (2015).
13. Salisbury, F.B. and Ross, C.W., Plant Physiology. 4th edition. *Wadsworth Publishing Company*. Belmont, California (1991).